Related Rates

Obj: Solve Related Rates Problems.
Related rate problems address how fast something is changing when another thing is changing at a known speed. This was one of the main reasons calculus was initially developed.

Two or more variables that vary with each other and vary with time are related rates of change. First we must differentiate with respect to time:

Equation	Derivative
1. $A = \pi r^2$	
2. $V = \pi r^2 h$	
3. $a^2 + b^2 = c^2$	
4. $A = s^2$	
5. $A = \frac{1}{2}bh$	

Ex. 1: Suppose *x* and *y* are both differentiable functions of *t* and are related by the equation $y = x^2 + 3$. Find $\frac{dy}{dt}$ when *x*= 1, given that $\frac{dx}{dt}$ = 2 when *x* = 1.

Guidelines for Solving Related Rate Problems

1. Draw a picture
2. Make a list of all known and unknown rates and quantities.
3. Label each quantity that changes with time.
4. Relate the variables in an equation
5. Differentiate with respect to time
6. Substitute the known quantities and rates and solve.

****Note rates will be negative if distance/quantity is shrinking

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<u>Ex 2.</u> Type 1 - Geometric formulas...the balloon problem

When a balloon is inflated the volume of the balloon increases with respect to time, as does the radius. Suppose the radius, r, is increasing at a rate of 2 in/min, Find the rate of change of the volume when the radius is 6 in.

<u>Ex. 3:</u> A pebble is dropped into a calm pond, causing ripples in the form of concentric circles. The radius *r* of the outer ripple is increasing at a constant rate of 1 foot per second. When the radius is 4 feet, at what rate is the total area *A* of the disturbed water changing?

<u>Ex. 4:</u> Joey is perched precariously at the top of a 10 foot ladder leaning against the wall of an apartment building when it starts to slide down the wall at a rate of 4 ft/min. Joey's friend Lou is standing on the ground 6 feet away from the wall. How fast is the base of the ladder moving when it hits Lou?

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Ex 5. Similar triangles and multiple variables.
A water tank in the form of a right circular cone with radius ,*R*, of 6′ and height,*H*, of 18′ is leaking water at the rate of 2 cu.ft./hr. What is the rate of change of *h* when *r*=4′?

Ex. 6: A television camera at ground level is filming the lift-off of a space shuttle that is rising according to the position equation $s = 50t^2$, where *s* is in feet and *t* is in seconds. The camera is 2000 feet from the launch. Find the rate of change in the angle of elevation of the camera 10 seconds after lift-off.

Ex 7.: Someone who is 6 feet tall is walking away from a lamp post at a rate of 5 feet per minute. The lamp post is 20 feet tall. The person casts a shadow on the ground in front of them. How fast is the shadow growing?